

## Test: Work &amp; Energy

Equations and Constants:

$$\bar{v} = \frac{\Delta x}{\Delta t}$$

$$v = \frac{dx}{dt}$$

$$\bar{a} = \frac{\Delta v}{\Delta t}$$

$$a = \frac{dv}{dt}$$

$$\bar{v} = \frac{1}{2}(v_i + v_f)$$

$$|g| = 10 \text{ m/s}^2$$

$$x = \frac{1}{2}at^2 + v_i t + x_i$$

$$v = at + v_i$$

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$R = \frac{v^2 \sin 2\theta}{g}$$

$$a_c = \frac{v^2}{r}$$

$$\sum F = ma$$

$$w = mg$$

$$w_{\perp} = mg \cos \theta$$

$$w_{\parallel} = mg \sin \theta$$

$$f = \mu N$$

$$W = Fd \cos \theta$$

$$W = \int \vec{F} \cdot d\vec{x}$$

$$K = \frac{1}{2}mv^2$$

$$P = \frac{dW}{dt}$$

$$P = Fv$$

$$U_{\text{gravity}} = mgh$$

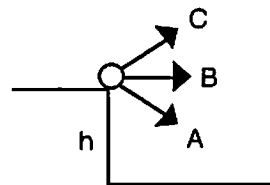
$$U_{\text{spring}} = \frac{1}{2}kx^2$$

**Multiple Choice:** Choose the letter of the best answer. 3 points each.Unless otherwise stated, ignore the effects of air resistance.  $|g| = 10 \text{ m/s}^2$ 

1. C A skier is moving at the top of a big hill and has a total mechanical energy of 10,000 J. She skies to the bottom of the hill. Friction dissipates 6000 J of energy. How much kinetic energy does the skier have at the bottom of the hill?  
 a. 10,000 J.      b. 16,000 J.      c. 4000 J.  
 d. It would depend on the hill she was on because we need to know how high the hill was and what the base angle was.
2. C A 2kg object has an initial kinetic energy of 50 J. Somehow, 25 J of work is done on it. What is its final speed?  
 a. 5 m/s.      b. 7.1 m/s.      c. 8.7 m/s.      d. 12.1 m/s.      e. 75 m/s.
3. C You are carrying a very heavy box across the room at constant speed. Why are you not doing any work on the box while carrying it?  
 a. Gravity is undoing the work you are doing.  
 b. The box is moving at constant speed.  
 c. You are pulling up vertically on the box while it is moving horizontally.  
 d. Of course you do work on the box because you get very tired.
4. D What happens to the "mechanical energy dissipated" by friction?  
 I. The object sliding gets warmer.  
 II. The internal energy of the object sliding increases.  
 III. It turns into potential energy.  
 a. I only.      b. II only      c. III only.      d. I & II only.      e. II & III only.

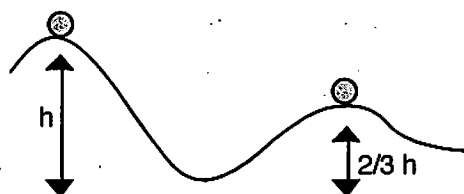
Problems 2 and 3 refer to the following:

A projectile is to be launched from the top of a cliff across a flat level field, as shown in the diagram to the right. It will be launched with one of the three initial angles shown, but the initial speed will always be the same.



5. E Which launch angle would result in the projectile going the fastest when it hits the ground?  
 a. A.      b. B.      c. C.      d. Whichever one goes the farthest.  
 e. They are all the same.
6. E Which launch angle would result in the greatest work done by gravity over the course of the flight?  
 a. A.      b. B.      c. C.      d. Whichever one goes the farthest.  
 e. They are all the same.

# Test: Work & Energy

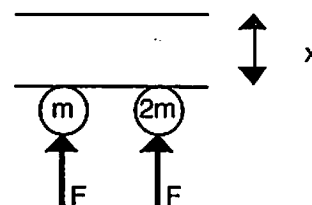


7. B An object is on a frictionless roller coaster shown, shown above. The object starts from the top of the first hill,  $h$ , with no initial speed. How fast is it going when it is at the top of the second hill?

a.  $\sqrt{\frac{1}{3}gh}$       b.  $\sqrt{\frac{2}{3}gh}$       c.  $\sqrt{gh}$       d.  $\sqrt{\frac{4}{3}gh}$       e.  $\sqrt{\frac{5}{3}gh}$

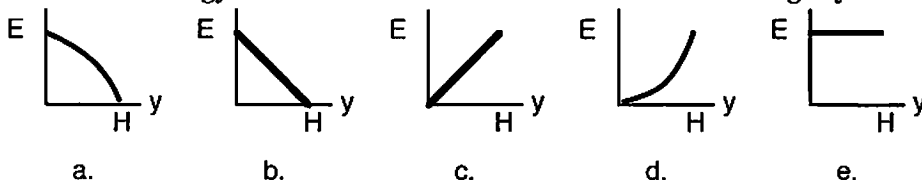
Problems 8 and 9 refer to the following:

Two objects,  $m$  and  $2m$ , are initially at rest and are pushed by identical forces  $F$  a distance  $x$  across a level frictionless table, as shown in the diagram to the right. They then slide across the table.

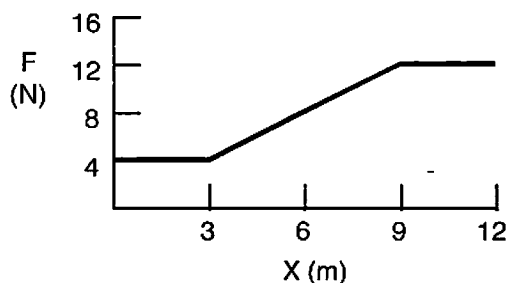


8. C Which mass has a greater kinetic energy at the end of the push?  
 a.  $m$ .      b.  $2m$ .      c. they are the same.      d. cannot tell.
9. A Which mass would experience a greater power?  
 a.  $m$ .      b.  $2m$ .      c. they are the same.      d. cannot tell.

10. E A ball is held at a height of  $H$  above a floor. It is then released and falls to the floor. If air resistance can be ignored, which of the following graphs best represents the total mechanical energy  $E$  of the ball while it falls as a function of height  $y$ ?

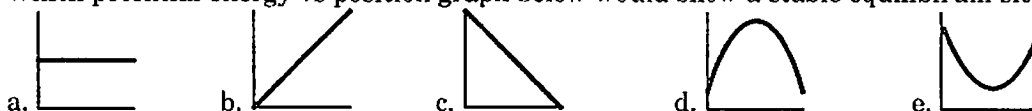


11. B A certain net force acts on a 5 kg object. The graph of the force versus the position of the object is shown in the diagram below. By how much does the kinetic energy of the object change between  $x = 3$  and  $x = 9$ ?



a. 12 J.      b. 48 J.      c. 60 J.      d. 72 J.      e. 108 J.

12. E Which potential energy vs position graph below would show a stable equilibrium situation?



## Test: Work &amp; Energy

**Problem Solving: Show all work.****Use work/energy principals!**

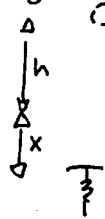
13. A 4 kg object sitting on a spring will compress the spring 15 cm. If the mass were held 30 cm above the uncompressed spring and let go, what would be the maximum compression of the spring?

$$mg = kx$$

$$4(10) = k(.15)$$

$$k = \frac{40}{.15}$$

$$k = 267$$



$$mg(h+x) = \frac{1}{2}kx^2$$

$$mgh + mgx = \frac{1}{2}kx^2$$

$$\frac{1}{2}kx^2 - mgx - mgh = 0$$

$$\frac{1}{2}(267)x^2 - 40x - 40(.3) = 0$$

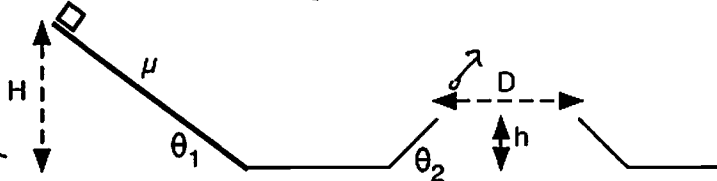
$$133x^2 - 40x - 12 = 0$$

$$x = \frac{40 \pm \sqrt{40^2 - 4(133)(-12)}}{2(133)}$$

$$x = \frac{40 \pm 89.4}{267}$$

$$x = .485 \text{ m}$$

14. An object is at the top of a small incline of height H. The incline has a coefficient of friction of  $\mu$  and abase angle of  $\theta_1$ . There is no more friction after the incline. After a short distance, there is a small ramp of base angle  $\theta_2$  and height h, still without friction. There is an identical ramp a distance D away from the first. What must be the height H in order for the object to just be able to land on the second ramp?



$$R = \frac{v^2 \sin 2\theta}{g}$$

$$v^2 = \frac{Rg}{\sin 2\theta}$$

$$v^2 = \frac{Dg}{\sin 2\theta_2}$$

$$mghH - \mu mg \cos \theta_1 \left( \frac{H}{\sin \theta_1} \right) = mgh + \frac{1}{2}mv^2$$

$$H - H \left( \frac{\mu \cos \theta_1}{\sin \theta_1} \right) = h + \frac{1}{2} \frac{D}{\sin 2\theta_2}$$

$$H(1 - \mu \cot \theta_1) = h + \frac{D}{2 \sin 2\theta_2} \Rightarrow$$

$$H = \frac{h + \frac{D}{2 \sin 2\theta_2}}{1 - \mu \cot \theta_1}$$

15. An object of mass m is pushed a distance x into a spring of spring constant k. It is then released, going over a frictionless hill of height and radius r. After the hill, there is a coefficient of friction of  $\mu$ , and the object slides to a stop in a distance d. It goes over the hill as fast as it can without becoming a projectile. What is the coefficient of friction?



$$\text{at top } mg = \frac{mv^2}{r}$$

$$v^2 = rg$$

$$\frac{1}{2}mv^2 + mgr - \mu mgd = 0$$

$$\frac{1}{2}rg + rg = \mu gd$$

$$\frac{3}{2}r = \mu d$$

$$\mu = \frac{3r}{2d}$$

$$\left( \text{or: } \frac{1}{2}kx^2 = \mu mgd \right)$$

$$\mu = \frac{kx^2}{2mgd}$$